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A New Class of Inhomogeneous Cosmological Models with Electromagnetic Field in Normal Gauge for Lyra's Manifold

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Abstract A new class of exact solutions of Einstein's modified field equations in inhomogeneous space-time for perfect fluid distribution with electromagnetic field is obtained in the context of normal gauge for Lyra's manifold. We have obtained solutions by considering the time dependent displacement field. The source of the magnetic field is due to an electric current produced along the z-axis. Only F_{12} is a non-vanishing component of the electromagnetic field tensor. It has been found that the displacement vector $\beta(t)$ behaves like the cosmological constant Λ in the normal gauge treatment and the solutions are consistent with the recent observations of Type Ia supernovae. Physical and geometric aspects of the models are also discussed in the presence of magnetic field.

Keywords Cosmology · Inhomogeneous models · Lyra's geometry · Electromagnetic field

1 Introduction

The inhomogeneous cosmological models play a significant role in understanding some essential features of the universe, such as the formation of galaxies during the early stages of evolution and process of homogenization. The early attempts at the construction of such

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models have been done by Tolman [1] and Bondi [2] who considered spherically symmetric models. Inhomogeneous plane-symmetric models were considered by Taub [3, 4] and later by Tomimura [5], Szekeres [6], Collins and Szafron [7, 8], Szafron and Collins [9]. Senovilla [10] obtained a new class of exact solutions of Einstein's equations without big bang singularity, representing a cylindrically symmetric, inhomogeneous cosmological model filled with perfect fluid which is smooth and regular everywhere satisfying energy and causality conditions. Later, Ruiz and Senovilla [11], Dadhich et al. [12], Patel et al. [13], Mehta and Gupta [14] and Pradhan et al. [15–18] have investigated inhomogeneous cosmological models in various contexts.

The occurrence of magnetic fields on a galactic scale is a well-established fact today, and its importance for a variety of astrophysical phenomena is generally acknowledged as pointed out by Zeldovich et al. [19]. Also Harrison [20] suggests that magnetic field could have a cosmological origin. As a natural consequences, we should include magnetic fields in the energy-momentum tensor of the early universe. The choice of anisotropic cosmological models in Einstein system of field equations leads to the cosmological models more general than Robertson–Walker model [21]. The presence of a primordial magnetic field in the early stages of the evolution of the universe is discussed by many authors [22–31]. Strong magnetic field can be created due to adiabatic compression in clusters of galaxies. A large-scale magnetic field gives rise to anisotropies in the universe. The anisotropic pressure created by the magnetic fields dominates the evolution of the shear anisotropy and decays slowly as compared to the case when the pressure is held isotropic [32, 33]. Such fields can be generated at the end of an inflationary epoch [34–38]. Anisotropic magnetic field models have significant contribution in the evolution of galaxies and stellar objects. Bali and Ali [39] obtained a magnetized cylindrically symmetric universe with an electrically neutral perfect fluid as the source of matter. Chakrabarty et al. [40], Pradhan and Ram [41] and Pradhan et al. [42–46] have investigated magnetized cosmological models in various contexts.

In 1917 Einstein introduced the cosmological constant into his field equations of general relativity in order to obtain a static cosmological model since, as is well known, without the cosmological term his field equations admit only non-static solutions. After the discovery of the red-shift of galaxies and explanation thereof Einstein regretted the introduction of the cosmological constant. Recently, there has been much interest in the cosmological term in the context of quantum field theories, quantum gravity, super-gravity theories, Kaluza–Klein theories and the inflationary-universe scenario. Shortly after Einstein's general theory of relativity Weyl [47] suggested the first so-called unified field theory based on a generalization of Riemannian geometry. With its backdrop, it would seem more appropriate to call Weyl's theory a geometrized theory of gravitation and electromagnetism (just as the general theory was a geometrized theory of gravitation only), instead a unified field theory. It is not clear as to what extent the two fields have been unified, even though they acquire (different) geometrical significance in the same geometry. The theory was never taken seriously in as much as it was based on the concept of non-integrability of length transfer; and, as pointed out by Einstein, this implies that spectral frequencies of atoms depend on their past histories and therefore have no absolute significance. Nevertheless, Weyl's geometry provides an interesting example of non-Riemannian connections, and recently Folland [48] has given a global formulation of Weyl manifolds clarifying considerably many of Weyl's basic ideas thereby.

In 1951 Lyra [49] proposed a modification of Riemannian geometry by introducing a gauge function into the structure-less manifold, as a result of which the cosmological constant arises naturally from the geometry. This bears a remarkable resemblance to Weyl's geometry. But in Lyra's geometry, unlike that of Weyl, the connection is metric preserving as in the Riemannian case; in other words, length transfers are integrable. Lyra also introduced the notion of a gauge and in the "normal" gauge the curvature scalar in identical to that of Weyl. In consecutive investigations Sen [50], Sen and Dunn [51] proposed a new scalar-tensor theory of gravitation and constructed an analog of the Einstein field equations based on Lyra's geometry. It is, thus, possible [50] to construct a geometrized theory of gravitation and electromagnetism much along the lines of Weyl's "unified" field theory, however, without the inconvenience of non-integrability length transfer.

Halford [52] has pointed out that the constant vector displacement field ϕ_i in Lyra's geometry plays the role of cosmological constant Λ in the normal general relativistic treatment. It is shown by Halford [53] that the scalar-tensor treatment based on Lyra's geometry predicts the same effects within observational limits as the Einstein's theory. Several authors Sen and Vanstone [54], Bhamra [55], Karade and Borikar [56], Kalyanshetti and Wagmode [57], Reddy and Innaiah [58], Beesham [59], Reddy and Venkateswarlu [60], Soleng [61], have studied cosmological models based on Lyra's manifold with a constant displacement field vector. However, this restriction of the displacement field to be constant is merely one for convenience and there is no a priori reason for it. Beesham [62] considered FRW models with time dependent displacement field. Singh and Singh [63–66], Singh and Desikan [67] have studied Bianchi-type I, III, Kantowaski–Sachs and a new class of cosmological models with time dependent displacement field and have made a comparative study of Robertson–Walker models with constant deceleration parameter in Einstein's theory with cosmological term and in the cosmological theory based on Lyra's geometry. Soleng [61] has pointed out that the cosmologies based on Lyra's manifold with constant gauge vector ϕ will either include a creation field and are equal to Hoyle's creation field cosmology [68–70] or contain a special vacuum field, which together with the gauge vector term, may be considered as a cosmological term. In the latter case the solutions are equal to the general relativistic cosmologies with a cosmological term.

Recently, Pradhan et al. [71–75], Casama et al. [76], Rahaman et al. [77, 78], Bali and Chandnani [79–81], Kumar and Singh [82], Singh [83], Rao, Vinutha and Santhi [84], Pradhan [85, 86] and Singh and Kale [87] have studied cosmological models based on Lyra's geometry in various contexts. With these motivations and following the technique of Bali and Singh [88], in this paper, we have obtained exact solutions of Einstein's modified field equations in inhomogeneous space–time within the frame work of Lyra's geometry in the presence of magnetic field for time varying displacement vector. This paper is organized as follows. In Sect. 1 the motivation for the present work is discussed. The metric and the field equations are presented in Sect. 2. In Sect. 3 the solutions of field equations for two cases are derived for time varying displacement field $\beta(t)$ in presence of magnetic field and their geometric and physical properties are also described. Finally, in Sect. 4 discussion and concluding remarks are given.

2 The Metric and Field Equations

We consider the metric in the form

$$ds^2 = dx^2 - dt^2 + B^2\,dy^2 + C^2\,dz^2, \tag{1}$$

where B and C are both functions of x and t. The energy-momentum tensor as taken has the form

$$T_i^j = (\rho + p)u_i u^j + p g_i^j + E_i^j, \tag{2}$$

where E_i^j is the electromagnetic field, given by

$$E_i^j = F_{il}F^{jl} - \frac{1}{4}F_{lm}F^{lm}g_i^j. \tag{3}$$

Here ρ and p are, respectively, the energy density and pressure of the cosmic fluid; F_{ij} is the components of electromagnetic field tensor; and u^i is the flow vector satisfying the condition

$$g_{ij}u^i u^j = -1. \tag{4}$$

The co-ordinates are considered to be co-moving so that $u^1 = 0 = u^2 = u^3$ and $u^4 = 1$. If we consider that the current flows along the z-axis, then F_{12} is the only non-vanishing component of F_{ij}.

The field equations (in gravitational units $c = 1$, $G = 1$), in normal gauge for Lyra's manifold, obtained by Sen [40] as

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -8\pi T_{ij}, \tag{5}$$

where ϕ_i is the displacement field vector defined as

$$\phi_i = \big(0, 0, 0, \beta(t)\big), \tag{6}$$

where other symbols have their usual meaning as in Riemannian geometry.

For the line-element (1), the field (5) with (2) and (6) lead to the following system of equations

$$-\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC} + \frac{B'C'}{BC} - \frac{3}{4}\beta^2 = 8\pi\left(p + \frac{F_{12}^2}{2B^2}\right), \tag{7}$$

$$-\frac{\ddot{C}}{C} + \frac{C''}{C} - \frac{3}{4}\beta^2 = 8\pi\left(p + \frac{F_{12}^2}{2B^2}\right), \tag{8}$$

$$-\frac{\ddot{B}}{B} + \frac{B''}{B} - \frac{3}{4}\beta^2 = 8\pi\left(p - \frac{F_{12}^2}{2B^2}\right), \tag{9}$$

$$-\frac{B''}{B} - \frac{C''}{C} - \frac{B'C'}{BC} + \frac{\dot{B}\dot{C}}{BC} + \frac{3}{4}\beta^2 = 8\pi\left(\rho + \frac{F_{12}^2}{2\bar{\mu}A^2B^2}\right), \tag{10}$$

$$\frac{\dot{B}'}{B} + \frac{\dot{C}'}{C} = 0. \tag{11}$$

Here, and also in the following expressions, a dot and a dash indicate ordinary differentiation with respect to t and x respectively.

From (7)–(9), we obtain

$$\frac{B'C'}{BC} - \frac{\dot{B}\dot{C}}{BC} = \frac{C''}{C} + \frac{\ddot{B}}{B}, \tag{12}$$

and

$$8\pi\frac{F_{12}^2}{B^2} = \frac{C''}{C} - \frac{\ddot{C}}{C} - \frac{B''}{B} + \frac{\ddot{B}}{B}. \tag{13}$$

The energy conservation equation $T^i_{i;j}=0$ leads to

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0, \tag{14}$$

and

$$\left(R^j_i-\frac{1}{2}g^j_i R\right)_{;j}+\frac{3}{2}(\phi_i\phi^j)_{;j}-\frac{3}{4}(g^j_i\phi_k\phi^k)_{;j}=0. \tag{15}$$

Equation (15) leads to

$$\begin{aligned}&\frac{3}{2}\phi_i\left[\frac{\partial\phi^j}{\partial x^j}+\phi^l\Gamma^j_{lj}\right]+\frac{3}{2}\phi^j\left[\frac{\partial\phi_i}{\partial x^j}-\phi_l\Gamma^l_{ij}\right]-\frac{3}{4}g^j_i\phi_k\left[\frac{\partial\phi^k}{\partial x^j}+\phi^l\Gamma^k_{lj}\right]\\&\quad-\frac{3}{4}g^j_i\phi^k\left[\frac{\partial\phi_k}{\partial x^j}+\phi_l\Gamma^l_{kj}\right]=0.\end{aligned} \tag{16}$$

Equation (16) is identically satisfied for $i=1,2,3$ but for $i=4$, it is reduced to

$$\frac{3}{2}\beta\dot{\beta}+\frac{3}{2}\beta^2\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0. \tag{17}$$

3 Solution of Field Equations in Presence of Magnetic Field

We have the five independent equations (7)–(11), in six unknowns B, C, ρ, p, β and F_{12}. For the complete determinacy of the system, we need one extra condition which is narrated hereinafter. The research on exact solutions is based on some physically reasonable restrictions used to simplify the field equations.

Let us consider functional separability of the metric coefficients as given by

$$B=f(x)g(t),\qquad C=h(x)k(t). \tag{18}$$

Equations (11) and (18) reduce to

$$\frac{f'/f}{h'/h}=-\frac{\dot{k}/k}{\dot{g}/g}=a\quad\text{(constant)}, \tag{19}$$

which leads to

$$\frac{f'}{f}=a\frac{h'}{h}, \tag{20}$$

and

$$\frac{\dot{k}}{k}=-a\frac{\dot{g}}{g}. \tag{21}$$

Equations (20) and (21) lead to

$$f=bh^a,\qquad k=dg^{-a}, \tag{22}$$

where b and d are constants of integrations. Using (18) in (12), we obtain

$$\frac{ah'^2}{h^2} - \frac{h''}{h} = \frac{\ddot{g}}{g} - \frac{a\dot{g}^2}{g^2} = \ell \quad \text{(say)}, \tag{23}$$

which gives

$$\frac{h''}{h} - \frac{ah'^2}{h^2} = -\ell, \tag{24}$$

and

$$\frac{\ddot{g}}{g} - \frac{a\dot{g}^2}{g^2} = \ell. \tag{25}$$

Here two possible cases arise.

3.1 Case I: when $a > 1$, $\ell > 0$

In this case (24) and (25) lead to

$$\begin{aligned} h &= K_2 \cosh^{-\frac{1}{\alpha\kappa}}(K_1 - \alpha x), & g &= K_4 \sec^{\frac{1}{\alpha\kappa}}(\alpha t + K_3), \\ f &= bK_2^k \cosh^{-\frac{a}{\alpha\kappa}}(K_1 - \alpha x), & k &= dK_4^{-k} \sec^{-\frac{a}{\alpha\kappa}}(\alpha t + K_3), \end{aligned} \tag{26}$$

where K_1, K_2, K_3, K_4 are integrating constants, $\kappa = \sqrt{\frac{a-1}{\ell}}$ and $\ell\kappa = \alpha$. Accordingly, we obtain

$$B = fg = bK_2^k K_4 \cosh^{-\frac{a}{\alpha\kappa}}(K_1 - \alpha x) \sec^{\frac{1}{\alpha\kappa}}(\alpha t + K_3), \tag{27}$$

and

$$C = hk = dK_2 K_4^{-k} \cosh^{-\frac{1}{\alpha\kappa}}(K_1 - \alpha x) \sec^{-\frac{a}{\alpha\kappa}}(\alpha t + K_3). \tag{28}$$

In this case, after suitable transformation of coordinates, the metric (1) reduces to the form

$$\begin{aligned} \mathrm{d}s^2 = \left(\mathrm{d}X^2 - \mathrm{d}T^2\right) &+ \cosh^{-\frac{2a}{\alpha\kappa}}(\alpha X) \sec^{\frac{2}{\alpha\kappa}}(\alpha T)\, \mathrm{d}Y^2 \\ &+ \cosh^{-\frac{2}{\alpha\kappa}}(\alpha X) \sec^{-\frac{2a}{\alpha\kappa}}(\alpha T)\, \mathrm{d}Z^2. \end{aligned} \tag{29}$$

3.1.1 *Some Physical and Geometric Properties of the Model in Presence of Magnetic Field*

Equation (17) gives

$$\frac{\dot{\beta}}{\beta} = -\left(\frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \quad \text{as } \beta \neq 0, \tag{30}$$

which leads to

$$\frac{\dot{\beta}}{\beta} = \frac{\alpha(1-a)}{\kappa} \tan(\alpha T). \tag{31}$$

Equation (31) on integration gives

$$\beta = \cos^{\frac{(a-1)}{\alpha\kappa}}(\alpha T). \tag{32}$$

Using (27), (28) and (32) in (7) and (10), the expressions for pressure p and density ρ for the model (29) are given by

$$8\pi p = \left(\frac{\alpha}{\kappa} + \frac{1}{\kappa^2}\right)\tanh^2(\alpha X) - \left(\frac{a^2}{\kappa^2} - \frac{a\alpha}{\kappa}\right)\tan^2(\alpha T) + \frac{(a-1)\alpha}{\kappa} - \frac{4\pi F_{12}^2}{\cosh^{-\frac{2a}{\alpha\kappa}}(\alpha X)\sec^{\frac{2}{\alpha\kappa}}(\alpha T)} - \frac{3}{4}\cos^{\frac{2(a-1)}{\alpha\kappa}}(\alpha T), \tag{33}$$

$$8\pi\rho = \frac{(a+1)\alpha}{\kappa} - \frac{a}{\kappa^2}\tan^2(\alpha T) - \left(n + \frac{a\alpha}{\kappa} + \frac{3a^2}{\kappa^2}\right)\tanh^2(\alpha X) - \frac{4\pi F_{12}^2}{\cosh^{-\frac{2a}{\alpha\kappa}}(\alpha X)\sec^{\frac{2}{\alpha\kappa}}(\alpha T)} + \frac{3}{4}\cos^{\frac{2(a-1)}{\alpha\kappa}}(\alpha T), \tag{34}$$

where $n = \frac{\alpha}{\kappa} + \frac{1}{\kappa^2} - \frac{a^2}{\kappa^2}$, $\kappa > 0$.

The non-vanishing component F_{12} of the electromagnetic field tensor F_{ij} is obtained from (13)

$$4\pi F_{12}^2 = \frac{\sec^{\frac{2}{\alpha\kappa}}(\alpha T)}{\cosh^{\frac{2a}{\alpha\kappa}}(\alpha X)}\left[\frac{2a\alpha}{\kappa} + \left(n - \frac{a\alpha}{\kappa}\right)\tanh^2(\alpha X) + \left(n + \frac{a\alpha}{\kappa}\right)\tan^2(\alpha T)\right]. \tag{35}$$

The component of charge current density is given by

$$J^2 = \frac{-\tanh(\alpha X)\,\mathrm{sech}^{\frac{1}{\kappa\alpha}}(\alpha X)\sec^{-\frac{a}{\kappa\alpha}}(\alpha T)}{\sqrt{(8\pi I)}}\left[\alpha(n\kappa - a\alpha)\,\mathrm{sech}^2(\alpha X) - I\right], \tag{36}$$

where

$$I = \left[\frac{2a\alpha}{\kappa} + \left(n - \frac{a\alpha}{\kappa}\right)\tanh^2(\alpha X) + \left(n + \frac{a\alpha}{\kappa}\right)\tan^2(\alpha T)\right].$$

Halford [52] has pointed out that the displacement field ϕ_i in Lyra's manifold plays the role of cosmological constant Λ in the normal general relativistic treatment. From (32), it is observed that the displacement vector $\beta(T)$ is a periodic function of time. Figure 1, for parameters $\alpha = 0.009$, $\kappa = 1$, $a = 1.009$, shows the plot of $\beta(T)$ versus T. From Fig. 1, it is observed that the displacement vector $\beta(T)$ is a decreasing function of time and it approaches to a small positive value at late time, which is corroborated with Halford as well as with the recent observations [89–93] leading to the conclusion that $\Lambda(T)$ is a decreasing function of T. It is observed from (34) that the energy density is also a decreasing function of time and it is positive under appropriate condition.

The expressions for the expansion θ, Hubble parameter H, shear scalar σ^2, deceleration parameter q and proper volume V^3 for the model (29) are given by

$$H = 3\theta = 3\left(\frac{1-a}{\kappa}\right)\tan(\alpha T), \tag{37}$$

$$\sigma^2 = \frac{(a^2 - a + 1)}{\kappa^2}\tan^2(\alpha T), \tag{38}$$

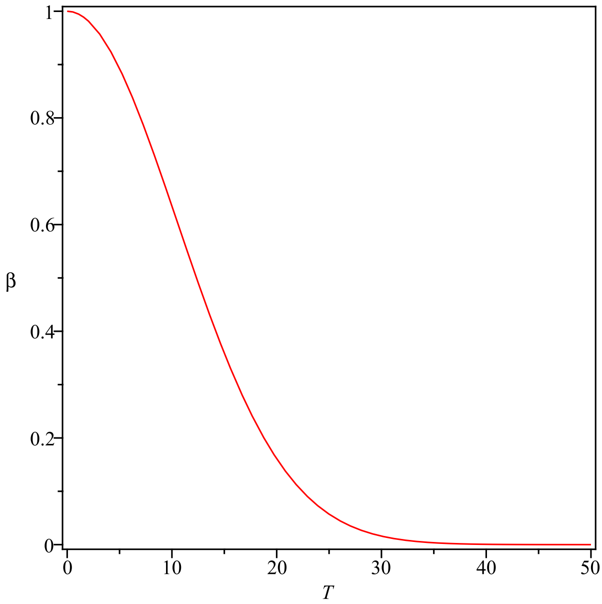

Fig. 1 Displacement vector $\beta(T)$ versus T

$$q = -1 - \frac{\alpha\kappa}{a-1}\operatorname{cosec}^2(\alpha T), \tag{39}$$

$$V^3 = \sqrt{-g} = \cosh^{-\frac{(a+1)}{\alpha\kappa}}(\alpha X)\sec^{-\frac{(a+1)}{\alpha\kappa}}(\alpha T). \tag{40}$$

From (37) and (38) we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{(a^2-a+1)}{(1-a)^2} = \text{constant}. \tag{41}$$

The rotation ω is identically zero.

The non-vanishing components of conformal curvature tensor are obtained as

$$\begin{aligned} C^{12}_{12} = \frac{1}{6}\bigg[&\left(\frac{3a^2}{\kappa^2} + \frac{2a\alpha}{\kappa} - \frac{a}{\kappa^2} - n\right)\tanh^2(\alpha X) \\ &+ \left(\frac{-a^2}{\kappa^2} + \frac{2a\alpha}{\kappa} - \frac{a}{\kappa^2} + n\right)\tan^2(\alpha T) + \frac{2\alpha}{\kappa}\bigg], \end{aligned} \tag{42}$$

$$\begin{aligned} C^{13}_{13} = C^{24}_{24} = \frac{1}{6}\bigg[&\left(\frac{a^2}{\kappa^2} - \frac{a\alpha}{\kappa} - \frac{a}{\kappa^2} + 2n\right)\tanh^2(\alpha X) \\ &+ \left(\frac{a^2}{\kappa^2} - \frac{a\alpha}{\kappa} - \frac{a}{\kappa^2} - \frac{2\alpha}{\kappa} - \frac{2}{\kappa^2}\right)\tan^2(\alpha T) - \frac{4\alpha}{\kappa}\bigg], \end{aligned} \tag{43}$$

$$\begin{aligned} C^{14}_{14} = C^{23}_{23} = \frac{1}{6}\bigg[&\left(\frac{2a}{\kappa^2} - \frac{a\alpha}{\kappa} - n\right)\tanh^2(\alpha X) \\ &+ \left(\frac{2a^2}{\kappa^2} - \frac{a\alpha}{\kappa} + \frac{2a}{\kappa^2} + n\right)\tan^2(\alpha T) + \frac{2\alpha}{\kappa}\bigg], \end{aligned} \tag{44}$$

$$C_{14}^{24} = \frac{a}{\kappa^2} \tanh(\alpha X) \tan(\alpha T), \tag{45}$$

$$C_{13}^{34} = -\frac{a}{\kappa^2} \tanh(\alpha X) \tan(\alpha T). \tag{46}$$

Generally the model (29) represents an expanding, shearing and non-rotating universe in which the flow vector is geodetic. The model (29) has initial singularity at $X = 0$, $T = 0$. The model will start expanding at $T > \frac{\pi}{\alpha}$ and the expansion will be maximum at $T = 0$, $T = \frac{\pi}{\alpha}$. Since $\frac{\sigma}{\theta}$ = constant, the model does not approach isotropy. As T increases the proper volume also increases. The physical quantities p and ρ decrease as F_{12} increases. It is observed from (39) that $q < 0$ when $\kappa\alpha > 0$, which implies an accelerating model of the universe. Recent observations of type Ia supernovae [89–93] reveal that the present universe is in accelerating phase and deceleration parameter lies somewhere in the range $-1 < q \leq 0$. It follows that our model of the universe is consistent with recent observations. When $\alpha = 0$, the deceleration parameter q approaches the value (-1) as in the case of a de Sitter universe. The space-time is non-degenerate Petrov-type I, in general.

3.2 Case II: when $a < 1$, $\ell < 0$

In this case (24) and (25) lead to

$$\begin{aligned} &h = c_2 \cosh^{\frac{1}{d}}(d\zeta x + c_1), \qquad g = b_2 \sec^{-\frac{1}{d}}(b_1 - d\zeta t), \\ &f = bh^a = bc_2^a \cosh^{\frac{a}{d}}(d\zeta x + c_1), \qquad k = dg^{-a} = db_2^{-a} \sec^{\frac{a}{d}}(b_1 - d\zeta t), \end{aligned} \tag{47}$$

where b_1, b_2, c_1, c_2 are constants of integration, $a - 1 = -d$, $\ell = -\ell$, $\zeta = \sqrt{\frac{\ell}{d}}$, $d > 0$. Accordingly, we obtain

$$B = fg = bb_2c_2^a \cosh^{\frac{a}{d}}(d\zeta x + c_1) \sec^{\frac{-1}{d}}(b_1 - d\zeta t), \tag{48}$$

and

$$C = hk = dc_2b_2^{-a} \cosh^{\frac{1}{d}}(d\zeta x + c_1) \sec^{\frac{a}{d}}(b_1 - d\zeta t). \tag{49}$$

In this case, after suitable transformation of coordinates, the metric (1) reduces to the form

$$\begin{aligned} ds^2 = {}& \left(dX^2 - dT^2\right) + \cosh^{\frac{2a\ell}{w^2}}(wX) \sec^{\frac{2\ell}{w^2}}(wT)\, dY^2 \\ &+ \cosh^{-\frac{2\ell}{w^2}}(wX) \sec^{-\frac{2a\ell}{w^2}}(wT)\, dZ^2, \end{aligned} \tag{50}$$

where $\zeta d = w$.

3.2.1 Some Physical and Geometric Properties of the Model in the Presence of a Magnetic Field

In this case (17) leads to

$$\frac{\dot{\beta}}{\beta} = \frac{\ell d\zeta(a-1)}{w} \tan(wT), \tag{51}$$

which on integration gives

$$\beta = \cos^{\frac{\ell(a-1)}{w^2}}(wT). \tag{52}$$

The expressions for pressure p and density ρ for the model (50) are given by

$$8\pi p = \left(\frac{\ell^2}{w^2} + \ell\right)\tanh^2(wX) - \left(\frac{a^2\ell^2}{w^2} - a\ell\right)\tan^2(wT) + (a-1)\ell$$
$$- \frac{4\pi F_{12}^2}{\cosh^{\frac{2a\ell}{w^2}}(wX)\sec^{\frac{2\ell}{w^2}}(wT)} - \frac{3}{4}\cos^{\frac{2\ell(a-1)}{w^2}}(wT), \tag{53}$$

$$8\pi\rho = (a+1)\ell - \frac{a\ell^2}{w^2}\tan^2(wT) - \left\{\left(a^2+a+1\right)\frac{\ell^2}{w^2} + (a+1)\ell\right\}\tanh^2(wX)$$
$$- \frac{4\pi F_{12}^2}{\cosh^{\frac{2a\ell}{w^2}}(wX)\sec^{\frac{2\ell}{w^2}}(wT)} + \frac{3}{4}\cos^{\frac{2\ell(a-1)}{w^2}}(wT). \tag{54}$$

The non-vanishing component F_{12} of the electromagnetic field tensor F_{ij} is obtained from (13)

$$4\pi F_{12}^2 = \cosh^{\frac{2a\ell}{w^2}}(wX)\sec^{\frac{2\ell}{w^2}}(wT)\left[2a\ell + \left\{(1+a)\ell + \left(1-a^2\right)\frac{\ell^2}{w^2}\right\}\right.$$
$$\left.\times\tan^2(wT) + \left\{(1-a)\ell + \left(1-a^2\right)\frac{\ell^2}{w^2}\right\}\tanh^2(wX)\right]. \tag{55}$$

The component of charge current density is given by

$$J^2 = \frac{-\tanh(wX)\operatorname{sech}^{-\frac{a\ell}{w^2}}(wX)\sec^{-\frac{2\ell}{w^2}}(wT)}{\sqrt{(8\pi\P)}}$$
$$\times\left[w\left\{(1-a)\ell + \left(1-a^2\right)\frac{\ell^2}{w^2}\right\}\operatorname{sech}^2(wX) - \frac{\ell\P}{w}\right], \tag{56}$$

where

$$\P = \left[2a\ell + \left\{(1+a)\ell + \left(1-a^2\right)\frac{\ell^2}{w^2}\right\}\tan^2(wT)\right.$$
$$\left. + \left\{(1-a)\ell + \left(1-a^2\right)\frac{\ell^2}{w^2}\right\}\tanh^2(wX)\right]. \tag{57}$$

From (52), it is observed that the displacement vector $\beta(T)$ is a periodic function of time. Figure 2, for the parameters $w = 0.005$, $\ell = -1$, $a = 0.92$, demonstrates the variation of $\beta(T)$ versus T. From Fig. 2, it is observed that the displacement vector $\beta(T)$ is a decreasing function of time and it approaches to a small positive value at late time, which is corroborated with Halford as well as with the recent observations [89–93] leading to the conclusion that $\Lambda(T)$ is a decreasing function of T. It is observed from (54) that the energy density is also a decreasing function of time and it is positive under appropriate condition.

The expressions for the expansion θ, Hubble parameter H, shear scalar σ^2, deceleration parameter q and proper volume V^3 for the model (50) are given by

$$H = 3\theta = 3\frac{(1-a)\ell}{w^2}\tan(wT), \tag{58}$$

$$\sigma^2 = \frac{(a^2-a+1)\ell^2}{w^4}\tan^2(wT), \tag{59}$$

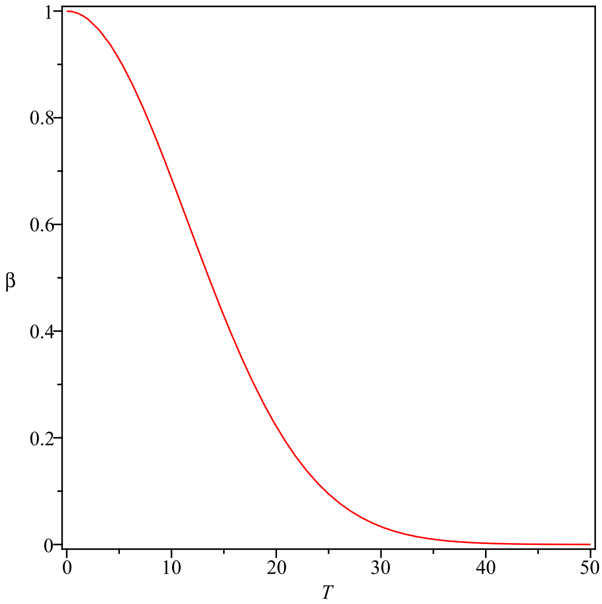

Fig. 2 Displacement vector $\beta(T)$ versus T

$$q = 1 + \frac{w^2}{\ell(a-1)}\operatorname{cosec}^2(wT), \tag{60}$$

$$V^3 = \sqrt{-g} = \cosh^{-\frac{\ell(a-1)}{w^2}}(wX)\sec^{-\frac{\ell(a-1)}{w^2}}(wT). \tag{61}$$

From (58) and (59) we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{(a^2-a+1)}{(1-a)^2} = \text{constant}. \tag{62}$$

The rotation ω is identically zero.

The non-vanishing components of conformal curvature tensor are obtained as

$$\begin{aligned} C^{12}_{12} = C^{34}_{34} = \frac{1}{6}\bigg[&\left(\frac{2a^2\ell^2}{w^2} - \frac{a\ell^2}{w^2} - \frac{\ell^2}{w^2} - (a+1)\ell\right)\tanh^2(wX) \\ &+ \left(\frac{\ell^2}{w^2} - \frac{2a^2\ell^2}{w^2} - \frac{a\ell^2}{w^2} + (2a+1)\ell\right)\tan^2(wT) + 2\ell\bigg], \end{aligned} \tag{63}$$

$$\begin{aligned} C^{13}_{13} = C^{24}_{24} = \frac{1}{6}\bigg[&\left(\frac{a^2\ell^2}{w^2} - \frac{a\ell^2}{w^2} - \frac{2\ell^2}{w^2} - (a+2)\ell\right)\tan^2(wT) \\ &+ \left(\frac{2\ell^2}{w^2} - \frac{a^2\ell^2}{w^2} - \frac{a\ell^2}{w^2}\right)\tanh^2(wX) - 2\ell\bigg], \end{aligned} \tag{64}$$

$$\begin{aligned} C^{14}_{14} = C^{23}_{23} = \frac{1}{6}\bigg[&\left(\frac{a^2\ell^2}{w^2} + \frac{2a\ell^2}{w^2} + \frac{\ell^2}{w^2} - (a-1)\ell\right)\tan^2(wT) \\ &- \left(\frac{\ell^2}{w^2} + \frac{a^2\ell^2}{w^2} - \frac{2a\ell^2}{w^2} + (a+1)\ell\right)\tanh^2(wX)\bigg], \end{aligned} \tag{65}$$

$$C^{24}_{14} = \frac{a\ell^2}{w^2} \tanh(wX) \tan(wT), \tag{66}$$

$$C^{34}_{13} = -\frac{a\ell^2}{w^2} \tanh(wX) \tan(wT). \tag{67}$$

Generally the model (50) represents an expanding, shearing and non-rotating universe in which the flow vector is geodetic. The model (50) has initial singularity at $X = 0$, $T = 0$. The model will start expanding at $T > \frac{\pi}{\alpha}$ and the expansion will be maximum at $T = 0$, $T = \frac{\pi}{w}$ and expansion will be maximum at $T = \frac{3\pi}{2w}$. The expansion stops at $T = 0$, $T = \frac{\pi}{w}$. Since $\frac{\sigma}{\theta}$ = constant, the model does not approach isotropy. As T increases the proper volume also increases. The physical quantities p and ρ decrease as F_{12} increases. It is observed from (60) that $q > 0$ always. So in this case the model is in decelerating phase. The space-time is non-degenerate Petrov-type I, in general.

4 Discussion and Concluding Remarks

By revisiting the solutions of Bali and Singh [88], in this paper, we have obtained a new class of exact solutions of Einstein's modified field equations for inhomogeneous space-time with a perfect fluid distribution within the framework of Lyra's geometry in presence of magnetic field. The solutions are obtained by using the functional separability of the metric coefficients. The source of the magnetic field is due to an electric current produced along the z-axis. F_{12} is the only non-vanishing component of electromagnetic field tensor. In both cases the electromagnetic field tensors are given by (35) and (55). It is observed that in the presence of magnetic field, the rate of expansion of the universe is faster than that in absence of magnetic field (although the results in absence of magnetic field are not reported in the present paper). The idea of primordial magnetism is appealing because it can potentially explain all the large-scale fields seen in the universe today, specially those found in remote proto-galaxies. As a result, the literature contains many studies examining the role and the implications of magnetic fields for cosmology. In the presence of a magnetic field both the models (29) and (50) represents an expanding, shearing and non-rotating universe in which the flow vector is geodetic.

In spite of homogeneity at large scale our universe is inhomogeneous at small scale, so physical quantities being position-dependent are more natural in our observable universe if we do not go to super high scale. This result shows this kind of physical importance. It is observed that the displacement vectors $\beta(t)$ in both cases coincide with the nature of the cosmological constant Λ which has been supported by the work of several authors as discussed in the physical behaviour of the model in previous section. In recent time Λ-term has attracted theoreticians and observers for many a reason. The nontrivial role of the vacuum in the early universe generates a Λ-term that leads to inflationary phase. Observationally, this term provides an additional parameter to accommodate conflicting data on the values of the Hubble constant, the deceleration parameter, the density parameter and the age of the universe (for example, see [94] and [95]). In recent past there has been an upsurge of interest in scalar fields in general relativity and alternative theories of gravitation in the context of inflationary cosmology [96–98]. Therefore the study of cosmological models in Lyra's geometry may be relevant for inflationary models. There seems to be a good possibility of Lyra's geometry to provide a theoretical foundation for relativistic gravitation, astrophysics and cosmology. However, the importance of Lyra's geometry for astrophysical bodies is still an open question. In fact, it needs a fair trial for experiment.

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